Eq. (26) expresses the shape of the crack by

$$(x^2/1^2)+(w^2/b^2) = 1 \tag{27}$$

which shows that the effect of the uniform pressure is to widen the crack into an elliptical crack.

Again substituting the expression, Eq. (25), into Eq. (24) we obtain

$$\sigma_{yy}(x, o, t) = p_0 H(t)\frac{d}{dx}[(x^2-1)^{1/2}-x] = p_0 H(t)\left[\frac{x}{(x^2-1)^{1/2}} - 1\right] \tag{28}$$

Numerical Results

At a particular instant of time the variation of $\sigma_{yy}(x,o)/p_0$ with x outside the crack is shown in the graph. The values of x are taken along x-axis whereas $\sigma_{yy}(x, o)/p_0$ varies along y-axis.

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Study of Methods for Modeling Centerline Mass Fraction Decay in Turbulent Jets

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Introduction

PREDICTING the concentration field of gases mixing with different molecular weights is important in providing design guidance in a number of engineering systems, e.g., diffusion type chemical lasers, rocket injectors, and scramjets. The concentration field can be characterized, in part, by specifying the rates at which the gases diffuse across the mixing zone (jet width growth) and the concentration decay in the streamwise direction. Universal jet width growth and centerline decay laws are known[1] for single stream jets and wakes of constant density for the similarity region, Fig. 1. It would be useful if similar laws were available for complex flows such as variable density two stream jets. Not only would they provide a description of the concentration field far downstream but they would also provide insight into the functional form of the turbulent transport coefficients which are used to model details of the flowfield.[2] References 3–6 have reported methods which propose to model and/or characterize centerline mass fraction decay in turbulent axisymmetric jets. The results of these investigations[3–6] have been heavily dependent on observations made of select data. The objective of this investigation was to examine these proposed models using most of the multispecies jet mixing data reported in the open literature to date.

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Index categories: Jets, Wakes and Viscid-Inviscid Flow Interaction; Subsonic and Supersonic Air-Breathing Propulsion.

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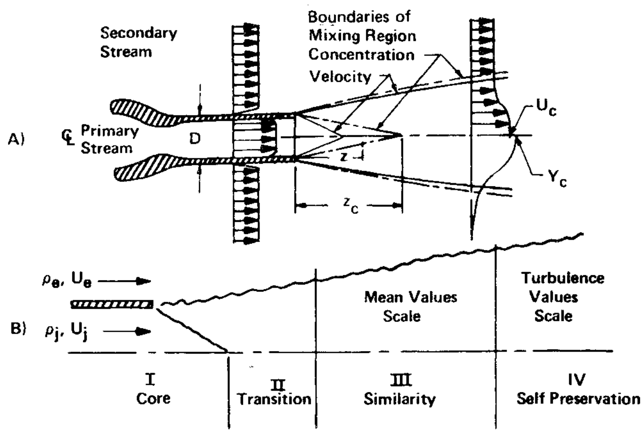

Fig. 1 Schematic of coaxial turbulent jet and definition of mixing regions.

Analysis

Zakkay et al.[3] have suggested that the centerline mass fraction may be determined from the relation

$$Y_c = (z/z_c)^{-m} \tag{1}$$

where $m = 2.0$ and z_c is the potential core length. Abromovich et al.[4] have suggested the same functional form as Eq. (1) but state $m = 1.7$ is the correct value. Schetz[5] has suggested that the decay exponent is correlatable with the ratio of the injected mass flux, $(\rho U)_j/(\rho U)_e$ where he showed

$$m = 2.0 \quad \text{for} \quad (\rho U)_j/(\rho U)_e < 1.0$$

and

$$m = 1.0 \quad \text{for} \quad (\rho U)_j/(\rho U)_e > 1.0$$

Cohen and Guile[6] state that the decay exponent for mass fraction is unity and independent of the density ratio, ρ_j/ρ_e if the velocity ratio U_j/U_e is near unity. Their results suggest $m = 1.0$ for $0.67 \lesssim U_j/U_e \lesssim 2.0$.

To study the reliability of these proposed models, 64 cases from nine different investigators[3,7–14] were analyzed to determine the value of the decay exponent. The data analyzed is presented in Table 1 where the values of $(\rho U)_j/(\rho U)_e$, U_j/U_e, m and the range covered by the data, $(\bar{z}_{max} - \bar{z}_{min})$, are listed.

The centerline mass fraction decay exponent was obtained by using a least squares fit of Eq. (1) and solving for z_c and m. (See Zelazny[15] for details.) This technique differed from past methods of obtaining z_c and m where a straight line was simply drawn through data on a log-log plot and its slope determined graphically. Using this different technique is significant since the difference between a decay exponent of $m = 1.5$ and 2.0 is only an angle of 7° when plotted on a log-log scale. In addition a judgment as to when to neglect points near the core region must be made when making these plots. This judgment is more quantitative using the least squares fit computer code, since it is required that the calculated values of z_c and m give a residual of less than 0.01. Consequently, points near the core are automatically deleted if including them resulted in residuals greater than 0.01.

Examination of the values of m showed that these values change significantly depending on the type of flow considered. Clearly, universal values of $m = 2.0$, Zakkay et al.,[3] or $m = 1.7$,

Table 1 Centerline mass fraction decay exponents vs ratios of jet to freestream velocity and mass flux

Investigator	$\frac{U_j}{U_e}$	$\frac{(\rho U)_j}{(\rho U)_e}$	$(\bar{z})_{min}$–$(\bar{z})_{max}$[a]	m
1) Chriss	6.3	0.56	7.0–14.6	1.89
2) Chriss	4.4	0.39	4.6–14.5	1.69
3) Chriss	3.8	0.32	6.6–20.8	1.60
4) Chriss	3.0	0.24	5.6–20.9	1.47
5) Chriss	2.4	0.19	4.8–12.8	1.57
6) Chriss	4.6	0.62	8.3–19.3	1.73
7) Chriss	3.2	0.41	4.6–16.3	1.56
8) Chriss	2.5	0.30	5.8–12.4	1.58
9) Zawacki and Weinstein	0.05	0.21	5.6–14.0	1.02
10) Zawacki and Weinstein	0.09	0.36	5.6–14.0	1.33
11) Zawacki and Weinstein	0.18	0.75	5.6–14.0	1.35
12) Zawacki and Weinstein	0.19	0.78	5.6–14.0	0.97
13) Zawacki and Weinstein	0.13	0.55	2.8–14.0	1.11
14) Zawacki and Weinstein	0.18	0.77	5.6–14.0	1.83
15) Zawacki and Weinstein	0.09	0.37	4.2–14.0	1.40
16) Zawacki and Weinstein	0.18	0.74	4.2–14.0	1.28
17) Zawacki and Weinstein	0.03	0.11	5.6–14.0	0.70
18) Zawacki and Weinstein	0.04	0.16	1.4–14.0	0.96
19) Zawacki and Weinstein	0.10	0.44	5.6–14.0	1.28
20) Zakkay et al.	1.46	0.047	16.7–30.0	1.91
21) Zakkay et al.	1.69	0.072	16.7–30.0	1.96
22) Zakkay et al.	2.42	0.124	16.7–30.0	2.05
23) Zakkay et al.	1.10	0.103	16.7–30.0	1.67
24) Zakkay et al.	1.67	0.185	16.7–30.0	1.84
25) Zakkay et al.	0.53	0.590	20.0–30.0	1.11
26) Zakkay et al.	0.56	0.790	20.0–30.0	1.69
27) Zakkay et al.	0.62	0.976	20.0–30.0	1.74
28) Alpinieri	0.67	0.040	5.25–10.0	1.77
29) Alpinieri	0.95	0.058	5.25–10.0	2.50
30) Alpinieri	1.25	0.076	5.25–10.0	1.96
31) Alpinieri	0.47	0.66	5.25–12.5	1.48
32) Alpinieri	0.65	0.95	7.5–12.5	1.48
33) Alpinieri	0.78	1.16	7.5–12.5	1.1
34) Forstall	5.0	5.0	24.0–135.0	1.07
35) Forstall	4.0	4.0	24.0–80.0	1.04
36) Forstall	2.0	2.0	24.0–133.0	1.13
37) Forstall	2.0	2.0		1.00
38) Forstall	1.33	1.33	56.0–135.0	1.45
39) Eggers and Torrence	0.74	0.64	25.0–49.0	1.91
40) Eggers and Torrence	0.65	0.55	44.0–114.0	2.30
41) Eggers and Torrence	0.81	0.79	42.0–85.0	2.48
42) Ragsdale et al.	1.00	5.00	7.0–16.3	0.47
43) Ragsdale et al.	0.80	3.90	9.3–14.0	0.85
44) Ragsdale et al.	0.60	2.90	4.7–16.3	0.54
45) Ragsdale et al.	0.23	1.10	9.3–16.3	1.38
46) Ragsdale et al.	0.71	3.40	9.3–16.3	0.92
47) Ragsdale et al.	0.58	2.80	9.3–16.3	0.86
48) Ragsdale et al.	0.48	2.30	4.7–16.3	0.90
49) Ragsdale et al.	0.35	1.70	4.7–16.3	0.92
50) Ragsdale et al.	0.29	1.40	7.0–16.3	0.80
51) Ragsdale et al.	0.23	1.10	2.3–14.0	0.93
52) Ragsdale et al.	0.13	0.61	4.7–16.3	0.87
53) Ragsdale et al.	0.08	0.40	2.4–16.3	0.98
54) Ragsdale et al.	0.17	0.82	2.3–16.3	0.76
55) Ragsdale et al.	0.36	1.70	4.7–16.3	1.60
56) Ragsdale et al.	0.11	0.53	9.3–16.3	0.74
57) Ragsdale et al.	0.08	0.41	9.3–16.3	0.96
58) Ragsdale et al.	0.06	0.27	2.3–11.7	0.69
59) D'Souza et al.	0.04	0.17	5.6–14.0	0.89
60) D'Souza et al.	0.11	0.48	2.8–14.0	1.0
61) D'Souza et al.	0.19	0.83	5.6–14.0	1.2
62) Keagy and Weller	∞ (N_2)	∞	12.0–49.0	1.07
63) Keagy and Weller	∞ (CO_2)	∞	15.0–48.0	1.20
64) Keagy and Weller	∞ (He)	∞	17.5–48.5	0.84

[a] $\bar{z} = z/D$

Abromovich et al.,[4] are not supported by these results. The value of m appears to be dependent on $(\rho U)_j/(\rho U)_e$, U_j/U_e and possibly on other parameters. In addition the effect of the experimental configuration may also play a role in determining the value of m, Ref. 4.

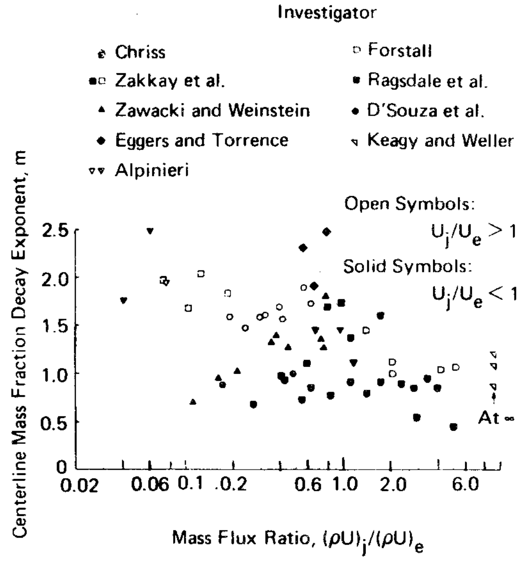

Fig. 2 Centerline mass fraction decay exponent vs flux ratio, $(\rho U)_j/(\rho U)_e$.

Figure 2 shows the values of m plotted against the mass flux ratio $(\rho U)_j/(\rho U)_e$; for clarity jetlike flows ($U_j/U_e > 1.0$) are represented by open symbols whereas wakelike flows ($U_j/U_e < 1$) are denoted as solid symbols. If only the jetlike flows are considered, these results show that indeed the trend observed by Schetz is reasonable with the exception that a more representative value of m for $(\rho U)_j/(\rho U)_e < 1$ appears to be closer to 1.7 than 2.0 as suggested by Abromovich et al.[4] On the other hand if both jet and wakelike data are considered, no discernible correlation between m and $(\rho U)_j/(\rho U)_e$ can be obtained. Considering only wakelike data interestingly suggests that maximum values of m are obtained near $(\rho U)_j/(\rho U)_e \approx 1.0$. A reason for such behavior is not obvious.

Figure 3 shows the values of m plotted against velocity ratio U_j/U_e; again for clarity, cases where $\rho_j/\rho_e \geqq 1.0$ are represented by open symbols whereas cases with $\rho_j/\rho_e < 1.0$ are designated by closed symbols. These results do not support the observation of Ref. 6 that m becomes unity over the range $0.67 \lesssim U_j/U_e \lesssim 2.0$.

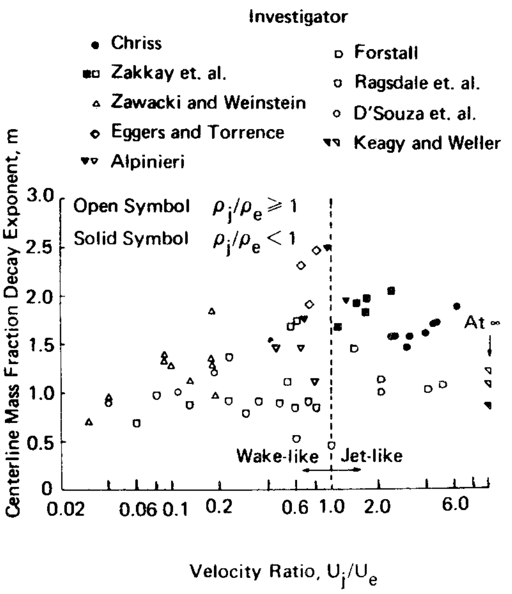

Fig. 3 Centerline mass fraction decay exponent vs velocity ratio U_j/U_e.

In trying to explain this discrepancy, it was noted (see Ref. 15 for details) that most of the data considered in Ref. 6 was either in the core or just downstream of the core region. Asymptotic decay exponents cannot be meaningfully defined in the transition region where m ranges from zero, just at the end of the core, to some finite value further downstream; therefore, values of m obtained from data for this region cannot be interpreted as meaningful asymptotic decay exponents.

Conclusions

The uncertainty involved in calculating the centerline mass fraction decay exponent as demonstrated from the preceding discussion leads to the following conclusions. Existing data does not support use of Eq. (1) to characterize centerline mass fraction decay with a universal decay exponent. Also the centerline mass fraction decay exponent cannot be correlated with the velocity ratio U_j/U_e or the mass flux ratio $(\rho U)_j/(\rho U)_e$ using existing experimental data. If such a correlation exists, data for the far downstream region covering a wide range of velocity and density ratios must be made available. Two facts which make such data difficult to obtain are: 1) in the far downstream regions the parameters being measured are small and hence subject to large experimental uncertainties, and 2) the simulation of an infinite external stream becomes increasingly difficult far downstream due to external influences, e.g., walls. Therefore, to obtain such data the use of superior diagnostic techniques and more sophisticated experimental facilities than used to date are required.

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Estimation of Turbulent Energy Dissipation Using Available Transfer Theories

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Introduction

THE closure problem of the turbulent kinetic energy equation in physical space requires the specification of the dissipation function ε as a function of the energy

$$\bar{e} = \sum_{i=1}^{3} \overline{u_i u_i}.$$

On dimensional considerations, Rotta[1] proposed the relation

$$\varepsilon = A(R)(\bar{e})^{3/2}/L \quad (1)$$

where L is the integral scale of the turbulence and $R = (\varepsilon\nu)^{1/4}L/\nu$. For sufficiently high Reynolds numbers, the function $A(R)$ becomes a universal constant. This constant value has been used by many authors to obtain solutions of turbulent boundary layer problems.‡ Near the wall, however, the value of R and Re become small and a constant value for $A(R)$ is not valid ($Re = (\bar{e})^{1/2}L/\nu$). Direct use of the energy equation near the wall requires a nonconstant formulation for $A(R)$.

Recently Trusov[4] obtained an expression which provides the necessary values of $A(R)$ based on consideration of the three-dimensional turbulent-energy spectral density function $E(k)$, where k is the wave-number, in the universal equilibrium range. Trusov shows impressive agreement with a wide variety of test data down to fairly low values of Reynolds number.§ Unfortunately, some of the test data used by Trusov do not provide measured values of the integral scale so that these data, as used by Trusov, may be subject to question. The deduced expression for $E(k)$ used by Trusov is essentially the same as that used by Pao.[5] Pao also provided comparative figures using a variety of experimental results. Reid[6] has also provided some comparative results using the formulations of the spectral density functions proposed by Heisenberg, Obukhoff and Kovasznay. Reid used the experimental data of Stewart and Townsend.[7]

The objective of this Note is to evaluate the previously referenced formulations of the turbulent energy spectral density function, including variations on Pao's expression, to obtain the function that seems to provide the best fit with the available test data over the full range of Reynolds numbers. The criteria used to evaluate the validity of the various density functions are a combination of those used by the previously referenced authors plus one additional consideration. The additional consideration is that the resulting values of $A(R)$ should approach a value of about 0.16 as the Reynolds number becomes very large. The value of 0.16 represents an average value not greatly different from the value used by Bradshaw[8] (0.1643), the value measured in wall turbulence by Lawn[9] (0.125), and the results of Rose[10] (0.17). This value is significantly less than the value of 0.314 obtained from the expression used by Beckwith and Bushnell.[11]

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Index category: Boundary Layers and Convective Heat Transfer—Turbulent.

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‡ There are a number of extensive bibliographies available on the subject of turbulent boundary-layer flows. For the present Note only two more recent review papers are pointed out, Refs. 2 and 3.

§ The range of Reynolds numbers shown in Ref. 4 extend from $R = 3$ ($Re \cong 4$) to $R = 500$ ($Re \cong 7630$).